

A black hole cast on a non-commutative background

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Abstract

In this work we describe a black hole, set on a non-commutative background. The model, which is relatively simple, is an exact solution of the Einstein Field Equations. Based on a proposition we put forward, we argue that introducing a matter density field on a non-commutative background sets up a mechanism that deforms the field into two distinct fields, one residing dominantly on the lattice tops (hereafter, on-cell) and the other residing dominantly in the inter-lattice regions (hereafter, off-cell). The two fields have different physical and thermodynamic characteristics which we describe, and some of which play a role in halting collapse to a singularity. For example, not surprisingly the on-cell (off-cell) fields manifest standard on-shell (off-shell) characteristics, respectively. Both the density and the net mass-energy are unchanged by the deformation mechanism. In our treatment the mass of a black hole defines its own size scale L of the interior region it occupies. Moreover, such a length is quantized, $L = 2N\sqrt{\theta}$, in terms of a minimum length scale $\sqrt{\theta}$. The approach has the advantage that the black hole density is not a function of the mass (as is the case in some recent treatments). The density is, instead a fixed quantity. As such, the approach puts an upper bound on black hole density, making it a universal parameter. The picture that emerges is that a black hole defined on a non-commutative background is both non-singular, holographic and quantized. Yet we also find, interestingly, that when taken over L the average value of the associated stress-energy tensor of the fields satisfies all classical energy conditions of GR.

1 Introduction

General Relativity (GR) rests on the proposition that spacetime and matter have an intertwined relationship; one in which matter instructs spacetime how to curve while spacetime tells matter how to move [1]. According to GR, under rather extreme conditions on their density, matter fields should induce curvature singularities on spacetime. Whether it is a (future-directed) black hole type or a (past-directed) cosmological type, a curvature singularity renders spacetime geodetically incomplete [2]. Thus with regard to the dynamics of matter fields, the initial singularity in cosmology would have constituted a "dead start" just as a black hole singularity would constitute a "dead end". Matter just can't easily wiggle its way out of a singularity! This concern has led to the view that under such extreme conditions, the breakdown is one of GR theory rather than one of spacetime. Approaches to rid

the black hole spacetime of physical singularities by modifying GR have a long history that goes back to Bardeen [3-11]. Lately, the issue (or non-issue) of the spacetime singularity is actively discussed in modern formulations of quantum gravity such as of String Theory [12] and Loop Quantum Gravity (LQG) [13]. In LQG, for example, the Ashtekar Hamiltonian [14], when quantized, leads to a minimum area a_0 [15] which, in turn, has led to a LQG inspired description of a non-singular Schwarzschild black hole interior [16-19] and black hole evaporation process [20]. The two theories are however still in evolution. For example, LQG still seeks a systematic approach to calculate scattering amplitudes and cross-sections needed for quantitative results while String Theory still seeks to extend the black hole description beyond near-extremal cases [21].

A potential ingredient of possibly any future theory of quantum gravity [22], [23] is the proposition that at very high (possibly Planckian) levels of resolution, the geometry of spacetime is inherently non-commutative (for a review see [21] and citations therein). In non-commutative gravity the coordinates x^μ become operators which obey some non-commutational relationship of the form $[x_\mu, x_\nu] = i\theta_{\mu\nu}$, where $\theta_{\mu\nu} = i\epsilon_{\mu\nu}\theta$ is a non-symmetric tensor with dimensions of length-squared. An uncertainty relationship of the form $|\Delta x_\mu \Delta x_\nu| \geq \frac{1}{2} |\theta_{\mu\nu}|$ is then implied and the coordinates have a fuzzy, smeared expectation value.

The concept of a quantized spacetime has been around for well over seventy years now [24]. Lately, an interest in non-commutative spacetimes has been rekindled by studies in String Theory [21]. In black hole physics, the idea that non-commutativity renders point-like objects to be smeared out has been explored to remove the would-be black hole singularity [25-32] and even to discuss gravastars [31]. In the more recent treatments the approach has been to apply a Voros map [34] on a Dirac delta function-like central density. This approach "softens" the otherwise singular density to a Gaussian form $\rho_\theta = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\theta}\right)$. The result is that the stress-energy tensor $T^{\mu\nu}$, and hence the spacetime curvature, is rendered finite everywhere in the black hole interior. In this respect the procedure just described has been successful with regard to regularization of matter distribution inside the black hole interior. There are however, in our view, some open questions:

(1) In the above Gaussian form of the density function the maximum density ρ_{\max} scales with the mass M . Noting that the black hole mass spectrum covers a large parameter space, with a conservative estimate of ~ 42 orders of magnitude (from say a $1g$ primordial black hole to a $10^{42}g$), this suggests an equally large spectrum in the central black hole density. Thus, in theory the density function could take on any values since neither theory nor observation puts a hard upper bound on black hole mass, yet. On the other hand, one would hope instead that a non-singular black hole should be defined by a universally fixed density parameter.

(2) A second question regards a technique whose application is not just restricted to the above-mentioned approach. It is often the case that in order to create a non-singular spacetime, one introduces fields whose stress-energy tensor $T^{\mu\nu} = \text{diag}[\rho, p_r, p_\perp, p_\perp]$ satisfies a radial equation of state of the form $p_r = -\alpha\rho$, where $\frac{1}{3} < \alpha \leq 1$. As is known such a construction will generally produce the desired negative pressure to off-set singularity formation. However, a couple of questions arise from here: (i) Does the entire fluid inside the black hole satisfy such an equation of state, as is often implied? And if so, is the *a priori* assumed dominance of gravity, and the implied outer Schwarzschild horizon, still justified?

Or if not, should one not make a clear statement regarding the state (dust, stiff...?) and the available fractional quantity of the matter fields, and how the interface of the implied 2-fluid is defined, if one exists? (ii) In the absence of a full quantum theory of gravity to deal with the issue definitely, one often introduces the above equation(s) of state by hand, and one often justifies the choice by the end result. It would help if a reasonably sound, albeit phenomenological, motivation for the choice of such equation(s) of state could be advanced in which one attempts to go a little beyond the "result-based" approach.

In this work, we seek to construct a non-singular black hole model (on a non-commutative background) that also addresses the above-mentioned questions, while not compromising the universality of the minimum length parameter $\sqrt{\theta}$. In our approach we introduce (with justification) a 2-fluid stress-energy tensor whose trace (curvature scalar) takes on different signs at different but specific points in the black hole interior region occupied by matter-energy. In general, the pressure components are non-vanishing at each point in the region occupied by matter fields. However as we show the average value of the individual pressure components across the region vanish so that only the density parameter contributes to the full mass-energy. As a result, the bulk matter interior of the black hole is shown to obey all the classical energy conditions, even though these conditions are violated in various specific regions in the same interior. Further, we will construct a black hole whose interior density is independent of the black hole mass but is instead a fixed universal parameter for the entire black hole mass-spectrum.

The rest of the paper is arranged as follows. In Section 2 we introduce matter fields on a non-commutative background. Section 3 addresses the two issues identified above for discussion and also presents the solution for the entire spacetime. Features of a black hole on a non-commutative background in our treatment are pointed out in Section 4. We conclude with a discussion in Section 5.

2 Matter fields on non-commutative background

A field $f(x)$, when defined on a non-commutative manifold suffers a deformation. Historically, there are two different prescriptions for defining the map onto such manifolds [34], namely the Wick-Voros product $f(x) \star_V g(x)$ and the Moyal product $f(x) \star_M g(x)$. On a complex plane $z_{\pm} = \frac{x \pm ix^2}{\sqrt{2}}$ the two products can be respectively defined by $f(x) \star_V g(x) =$

$\sum_n \left(\frac{\theta^n}{n!}\right) \partial_+^n f \partial_-^n g = f e^{\theta \overleftarrow{\partial}_+ \overrightarrow{\partial}_-} g$ and $f(x) \star_M g(x) = f e^{\frac{1}{2}\theta \left(\overleftarrow{\partial}_+ \overrightarrow{\partial}_- - \overleftarrow{\partial}_- \overrightarrow{\partial}_+\right)} g$, where $\partial_{\pm} = \frac{\partial}{\partial z_{\pm}}$. While the two maps do appear different, it has been shown [34] that the resulting S matrix from both prescriptions is essentially the same.

In this work we take the matter fields towards the end of gravitational collapse to constitute a homogeneous fluid condensate. We then point out how the effects of non-commutativity will halt collapse to avoid formation of a spacetime singularity. The resulting density field ρ will be shown to be independent of the black hole mass M . As a consequence the approach may be used to describe the interior of a black hole of any mass M in a consistent way and at the same time to establish the maximum density as a universal parameter.

The problem, here, reduces to one of projecting a homogeneous density field ρ_0 onto a spherical non-commutative manifold in a region \mathcal{R} of size L and inter-lattice interval $2\sqrt{\theta}$, corresponding to a lattice cell of size $\sqrt{\theta}$. In this work we will restrict the effects of non-commutativity in the radial direction. We begin by finding it convenient to write the density as a scalar product of a complex field $\rho_0 = \phi\phi^*$, which when mapped onto a non-commutative manifold deforms to $\phi(z_+) \star \phi^*(z_-)$. Independent of the map used (Moyal or Voros) it is easily seen that inducing a constant, homogeneous density field onto a non-commutative background reproduces the constant field,

$$\phi(z_+) \star_V \phi^*(z_-) = \phi(z_+) \star_M \phi^*(z_-) = \phi\phi^* = \rho_0. \quad (2.1)$$

At first, it would appear as if the mapping exercise above has no effect, since indeed the density ρ_0 remains unchanged. However, such an appearance would be deceptive. To see why, consider the following questions: (i) What are all the expected effects of a map on a non-commutative background? (ii) Is the resulting field still homogeneous, for example? In general based on the operator character of the map (Moyal or Voros, see above) a field (or fields) introduced onto a non-commutative manifold (1) can (self)-interact and/or (2) interact with the manifold on which they are induced. In our case the initial field is homogeneous and therefore will not self-interact (as Eq. 2.1 shows). However, it will expectedly interact with (*as it seeks to conform to*) the non-commutative background on which it is projected. This latter effect, (to be discussed below) is not manifest in Eq. 2.1.

Lemma 1 *A non-commutative spacetime manifold contains lattice (on-cell) and inter-lattice (off-cell) regions. On-cell/(off-cell fields) manifest characteristics reminiscent of on-shell/(off-shell) fields, respectively (with respect to their motion equations).*

Proposition 2 *In order to conform with its background, a homogeneous field mapped on a non-commutative spacetime manifold will undergo an internal deformation resulting into two different inhomogeneous fields. The deformation mechanism restricts regular matter fields to reside dominantly on-cell and non-matter field to reside dominantly off-cell. The deformation mechanism leaves the net density invariant.*

We will be guided by the Lemma and Proposition statements above in modeling features of a black hole interior. Note, however, the constraint demanded by Eq.1 that at any point in \mathcal{R} the net density must still be point-wise ρ_0 . We proceed by noting that one can write the complex field ϕ in terms of the lattice spacing, on which it is projected, as $\phi = \phi_0 e^{ik|z|}$ and $\phi^* = \phi_0 e^{-ik|z|}$, where we identify $k = \frac{\pi}{2\sqrt{\theta}}$, and where $2\sqrt{\theta}$ is the inter-lattice interval corresponding to a cell size $\sqrt{\theta}$. This can simply be rewritten as $\phi\phi^* = (\phi_0)^2 [\cos k|z| + i \sin k|z|] [\cos k|z| - i \sin k|z|]$, so that

$$\rho_0 = \rho_0 \sin^2 k\xi + \rho_0 \cos^2 k\xi, \quad (2.2)$$

with $\xi = |z|$ and with ρ_0 the density field on the non-commutative back ground given by Eq. 2.1. The relation in Eq. 2.2 is certainly a trivial mathematical identity which however demands a non-trivial physical interpretation.

We interpret the above result in the following manner. First, we consider the non-commutative region \mathcal{R} in which the fields reside to be a spherical lattice of radial size $L = 2N\sqrt{\theta}$ where $N = 1, 2, \dots$. By the Proposition, the map $\phi(z_+) \star \phi^*(z_-)$ can be viewed to constitute a mechanism through which the original matter density field deforms over the region \mathcal{R} into two physically different fields, which we denote as ρ_m and ρ_d . We recognize the two terms on the right hand side of Eq. 2.2 to represent the two fields and assign

$$\begin{aligned}\rho_m &= \rho_0 \sin^2 k\xi, \\ \rho_d &= \rho_0 \cos^2 k\xi.\end{aligned}\tag{2.3}$$

The field ρ_m resides predominantly on-cell and is therefore, by the Proposition, a regular matter density field. The field ρ_d resides predominantly off-cell. Note, however, that at any point in the black hole region \mathcal{R} the net density ρ , which satisfies $\rho_0 = \rho_m + \rho_d$, is still manifestly constant after the map on the non-commutative background (consistent with Eq. 2.1). We make the justifiably common choice that the matter field is stiff, obeying an equation of state $p_m(\rho_m) = \rho_m$. Based on this choice of matter fields equation of state and based on the off-shell character of ρ_d implied by the Proposition, and based on (later) considerations of energy conservation, one is left the only alternative that the off-cell density field ρ_d need obey an equation of state of the form $p_d(\rho_d) = -\rho_d$. This latter field is therefore de Sitter-like¹. We emphasize that this result, that the off-cell field be de Sitter-like, is not merely introduced by hand (as is usually the case), but is instead a consequence of the non-commutative deformation mechanism that we propose. We therefore have, on use of Eq. 2.2, that in region \mathcal{R} the respective pressure terms are given as,

$$\begin{aligned}p_m(\rho_m) &= \rho_0 \sin^2 k\xi, \\ p_d(\rho_d) &= -\rho_0 \cos^2 k\xi.\end{aligned}\tag{2.4}$$

The results in Eq. 2.4 imply that at any point in region \mathcal{R} the net pressure in the ξ -direction, $p = p_m + p_d$, associated with an inhomogeneous 2-fluid with a net density given by Eq. 2.2 satisfies an equation of state,

$$p_\xi = -\rho_0 \cos 2k\xi.\tag{2.5}$$

These results are represented in Figures 1 and 2, for an exemplary black hole of size $L = 6\sqrt{\theta}$ or $N = 3$. Figure 1 displays the density fields ρ_m and ρ_d and the net density $\rho = \rho_m + \rho_d$. Figure 2 displays the corresponding pressures p_m , p_d and the net pressure $p = p_m + p_d$. Note that while the net density is generally constant (except at the outermost edge), the net pressure at any point is a function of the coordinate ξ .

In the tangential directions, the pressure p_\perp of the isentropic fluid satisfies an equation of state $p_\perp = p_\xi - \frac{1}{2}\xi\partial_\xi p_\xi$ which in this case is given by $p_\perp = \rho_0 [\xi k \sin 2k\xi - \cos 2k\xi]$. We find it useful in our treatment to relax this condition to $p_\perp = p_\xi - \frac{1}{2}\xi\partial_\xi \rho_\xi$.

In this form the condition leaves the stress-energy tensor isotropic, since $\partial_\xi \rho_\xi = 0$. Notice though that while the pressure is isotropic, the 2-fluid is still inhomogeneous. The stress-energy tensor $T^{\mu\nu} = \text{diag}[\rho_0, p_\xi, p_\perp, p_\perp]$ for this spacetime then takes the form

$$T^{\mu\nu} = \text{diag}[\rho_0, -\rho_0 \cos 2k\xi, -\rho_0 \cos 2k\xi, -\rho_0 \cos 2k\xi].\tag{2.6}$$

¹We call the field de Sitter-like because, as pointed out later, it does not possess the standard symmetry requirements of the de Sitter group.

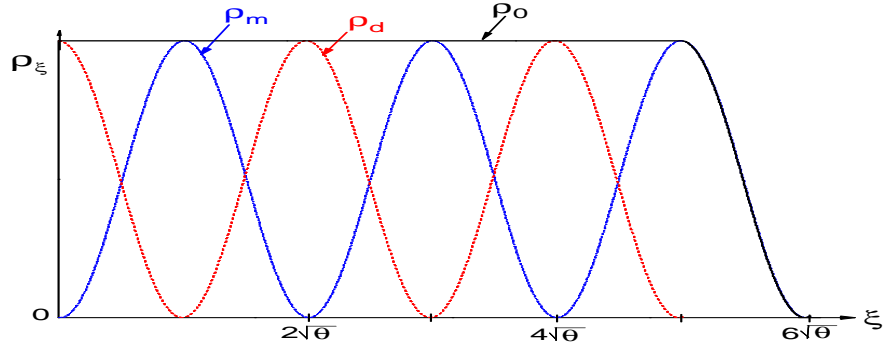


Figure 1: The density components (check labels): ρ_m of the matter fields, ρ_d of the de Sitter-like field, and ρ_0 the net density in the interior region \mathcal{R} of a size $N = 3$ black hole.

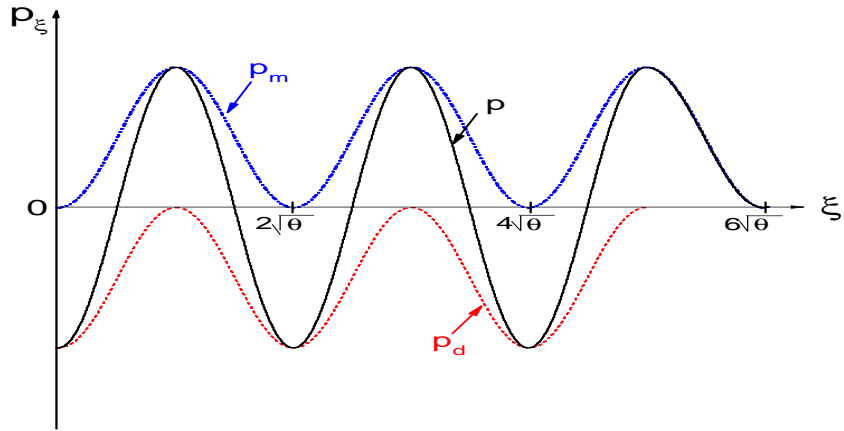


Figure 2: The pressure components (check labels): p_m of the matter fields, p_d of the de Sitter-like field, and p the net pressure in the ξ direction in the interior region \mathcal{R} of a size $N = 3$ black hole

3 The Spacetime

3.1 Matter distribution, and the density parameter

A major motivation of this work has been to discuss how matter may be distributed inside a black hole. In particular, the two issues were to construct a reasonably justified stress-energy tensor of the fields and also to make the black hole density a fixed parameter by making it independent of the mass. In this sub-section we briefly discuss these two issues, before presenting the spacetime solution.

The interior sub-region \mathcal{R} , where the fields reside, constitutes bands of concentric spheres of thickness $\sqrt{\theta}$ alternatingly dominated by matter and de Sitter-like fields, respectively. Each matter fluid region always encloses a de Sitter-like fluid region. The alternating set-up goes all the way down to the innermost central sphere $0 \leq \xi \leq \frac{1}{2}\sqrt{\theta}$, which sphere (see Figure 1 and 2) is de Sitter-like. At any 2-surface ξ in the region $0 \leq \xi \leq (L - \sqrt{\theta}) \approx L$ the density of the black hole has a constant value ρ_0 . Note that for most black holes $L \gg \sqrt{\theta}$ (the smallest black hole, $N = 1$, $L = 2\sqrt{\theta}$) so that for the purposes of discussing the density profile, the edge effects can be neglected and virtually $\rho(\xi) = \rho_0$ for $0 \leq \xi \leq L$. Further, we have shown that the negative pressure (at various specific values of ξ) responsible for halting the formation of a curvature singularity is produced through the non-commutative deformation mechanism which we proposed. As is seen in Figure 2 the net pressure oscillates between positive and negative values due to the on-cell matter fields and off-cell de Sitter-like field. Moreover, across the region \mathcal{R} occupied by the matter fields (see Figure 2) the average value of the pressure elements $\langle p_\xi \rangle$ (and hence also of $\langle p_\perp \rangle$) is a vanishing quantity when taken over the entire L . Consequently, the only global contribution of the stress-energy tensor to the total energy is that coming from the density parameter ρ_0 and given by

$$\langle T^\mu_\nu \rangle = \text{diag} [-\rho_0, 0, 0, 0] \quad (3.1)$$

This result in Eq.3.1 implies that the average value of the stress-energy tensor of this spacetime (and hence the bulk of matter in the interior black hole region \mathcal{R}) obeys all the classical energy conditions of GR, while Eq. 2.6 shows that specific surfaces in this region do not. In this way the non-commutativity driven local quantum behavior gives way to global classical behavior. The discussion here, along with our previous construction of the energy-stress tensor (which was motivated by the proposed deformation mechanism) addresses one of the issues we set out to deal with, namely the issue of the choice of the stress-energy tensor and the associated equation(s) of state.

That the only global contribution of the stress-energy tensor to the total energy is that coming from the density parameter ρ_0 implies that the deformation of the matter fields by the non-commutative background is, in this sense, energy conserving and allows one to recover the original mass as $M = 4\pi \int_0^L \rho_0 \xi^2 d\xi$ or

$$M = \frac{4}{3}\pi\rho_0 N^3 \theta^{\frac{3}{2}}. \quad (8)$$

This mass M is spread in the region \mathcal{R} bounded by a 2-surface of size $L^2 = (2N\sqrt{\theta})^2$ where $N = 1, 2, \dots$ is the only variable in the relation. Clearly the mass defines its size $L = L(M)$. In

turn this implies that the density ρ_0 is independent of the mass, and is in our case constant. We can thus treat ρ_0 as an upper bound of any matter density due to gravitational collapse, and therefore as a universal parameter. The result (Eq. 3.3) departs from results in which the black hole of any mass is contained inside a fixed area θ . Consequently, in our work, there is no degeneracy in mass-confinement of a black hole. This feature that a black hole determines its length size, with the result that the black hole density is a universal parameter, is the last of the two issues we set out to address.

Before leaving this discussion on matter distribution inside a black hole, it is useful to point out how the two fields which constitute the matter-energy behave at both boundaries $\xi = 0$ and $\xi = L$, and how this behavior affects the solution. As both Eqs. 2.2-2.5 and Figure 2 show, at the center $\xi = 0$, the matter fields vanish and the de Sitter-like field maximizes, to pure de Sitter, with a fixed density ρ_0 and pressure $-\rho_0$. It is the negative pressure in this innermost region $0 \leq \xi \leq \sqrt{\theta}$ that halts the immediate outlying spherical matter region $\sqrt{\theta} \leq \xi \leq 2\sqrt{\theta}$ from forming a curvature singularity at the center. On the other hand, at the boundary $\xi = L$ of \mathcal{R} the fields vanish in a well-behaved manner (in the mathematical sense) and for $\xi \geq L$ no such fields exist. This feature implies smooth continuity of the metric across the matter vacuum boundary, as demanded by the Israel junction conditions [36] [35].

3.2 The spacetime metric

The general region of a black hole inside the horizon is a Kantowski-Sachs spacetime, in which the radial and temporal coordinates switch their roles so that $r \rightarrow t$ and $t \rightarrow r$. As a result the variable $\xi = |z|$ in the preceding equation can now be identified (for $0 \leq \xi < 2M$) as a temporal variable $\xi = t$, where $t = 0$ is measured from the "center" of the spherical geometry of the spacetime.

In the sub-region \mathcal{R} the fields at any 2-surface ξ are described by the stress-energy tensor $T^\mu_\nu = \text{diag}[-\rho_0, p_\xi, p_\perp, p_\perp]$ given by Eq. 2.6. On solving the Einstein Field Equations [34] subject to Eq. 2.6 one finds the region \mathcal{R} to admit a metric line element,

$$ds^2 = - \left(1 - \frac{2m(\xi)}{\xi}\right)^{-1} d\xi^2 + \left(1 - \frac{2m(\xi)}{\xi}\right) d\xi^2 + \xi^2 d\Omega^2, \quad (3.3)$$

where $m(\xi)$ is the mass enclosed by the coordinate ξ in the region $0 \leq \xi \leq L$. At the matter-vacuum inter-surface $\xi = L$, $m(\xi) \rightarrow M$.

The outer interior sub-region $L \leq \xi \leq 2M$ which is still Kantowski-Sachs satisfies the vacuum Einstein equations to give a line element

$$ds^2 = - \left(1 - \frac{2M}{\xi}\right)^{-1} d\xi^2 + \left(1 - \frac{2M}{\xi}\right) dr^2 + \xi^2 d\Omega^2. \quad (3.4)$$

One notes that the functions representing the fields and their derivatives vanish at the 2-surface $\xi = L$, and are therefore well-behaved thereat. Such a feature guarantees the smooth matching of the metrics given by Eqs. 3.3 and 3.4 as they satisfy the Israel junction conditions [35], [34].

Finally at the outer, or Schwarzschild horizon $\xi = 2M$, the spatial and temporal coordinates switch and for $2M < r < \infty$ we recover the exterior vacuum Schwarzschild solution,

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \quad (3.5)$$

4 Features

In the previous section we have discussed two features of the black hole in our model, namely the local (quantum) and global (classical) behavior of the fields in the energy-stress tensor and the mass independence of the density parameter. In this section we briefly point out more features of the black hole in this model that we consider essential.

4.1 A black hole is a quantized, non-singular object

We have seen that the matter inside a black hole is distributed in a region of size $L = 2N\sqrt{\theta}$, $N = 1, 2, \dots$. This length size L is manifestly quantized in $\sqrt{\theta}$, as a result of non-commutative considerations. Thus, in this model, a black hole interior constitutes bands of quantized concentric spheres, each of average thickness $\sqrt{\theta}$, alternately dominated by matter and de Sitter-like fields, respectively. Each matter fluid region always encloses a de Sitter-like fluid region. The alternating set-up goes all the way down to the innermost central sphere. This last innermost sphere is (see Figure 1 and 2) manifestly de Sitter-like. The tendency for a given spherical band of matter fluid to collapse under its gravitational influence is therefore always counter-balanced by the negative pressure of the bounding inner sphere of de Sitter-like fluid. The result is that formation of the central singularity due to gravitational collapse is avoided. Based on this set-up, nature appears to protect spacetime from formation of curvature singularities. This feature is consistent with a conjecture that we previously put forward [37].

4.2 Trapped surfaces and horizons

The region \mathcal{R} consists of a system of nested trapped and untrapped surfaces. For example $\xi = L$, the outermost matter surface closest to the Schwarzschild horizon, is a trapped surface. On the other hand the central point $\xi = 0$ which lies on a de Sitter space (see Eqs. 2.2-2.5) is untrapped. Based on this and considering, as a simple example, the spacetime of the smallest black hole ($L = 2\sqrt{\theta}$, $N = 1$) of some mass M . Such a spacetime will have one region that constitutes a set of trapped surfaces bounded from out by $\xi < 2M$, and one inner region of untrapped surfaces that includes the black hole center. In between, the matter density pick and the adjacent de Sitter-like density pick at the center there exists some marginally trapped surface centered at $\xi = 0$ that divides the trapped and untrapped regions. Such a surface is a horizon. Thus the smallest black hole will have two horizons, including the Schwarzschild horizon. For now we will defer a detailed analysis of the horizon structure of this spacetime for a future discussion.

4.3 A black hole is holographic

The picture of a black hole interior that has emerged so far is one of a system of concentric spherical bands of matter fields interjected by concentric spherical bands of de Sitter-like regions. This quantized onion-like system goes in all the way down to the center, with the central region being de Sitter-like (see Eqs. 2.2-2.5 and Figures 1-3). Figures 1 and 2 show that the surface $\xi = (N - \frac{1}{2}) 2\sqrt{\theta}$ is special. We will call this 2-surface Σ_s . Surface Σ_s is the boundary inside which both (i) the black hole density is constant and maximum $\rho = \rho_0$, and (ii) the entire de Sitter-like fluid is confined. We consider the 2-fluid of matter and de-sitter-like field inside Σ_s to constitute a frozen system, in the following sense. We suppose that during gravitational collapse the fields crossing Σ_s will deposit all their dynamical degrees of freedom in the thin sub-region $(N - \frac{1}{2}) 2\sqrt{\theta} \leq \xi \leq N2\sqrt{\theta}$ overlying Σ_s , which we will call sub-region Δ . Consequently, the 2-fluid inside Σ_s is a super-frozen condensate with non-existent dynamical degrees of freedom and hence with zero entropy.

One recognizes sub-region Δ overlying the condensate system to be the only region (i) made purely of matter fields (ii) with a net density ρ that monotonically decreases to zero and (iii) with a free boundary at $L = N2\sqrt{\theta}$. The free outer boundary character of sub-region Δ facilitates the existence of quantum fluctuations which we suppose dominate this region. Thus, based on this argument, all dynamical degrees of freedom of the matter fields in a black hole will reside in sub-region Δ . It is important to recognize that in the ξ direction the sub-region Δ where quantum fluctuations are likely to dominate has a thickness, or more precisely a skin-depth, of size $\sqrt{\theta}$. Recall that the smallest length measure allowed by the uncertainty relation suggested by non-commutative gravity is $\left[\sqrt{|\Delta x_\mu \Delta x_\nu|} \geq \sqrt{\frac{1}{2} |\theta_{\mu\nu}|} \right]$. Based on this, it is clear that sub-region Δ cannot be distinguished from the surface $L = 2\sqrt{\theta}$. As a result we find that in our treatment the dynamical degrees of freedom of a black hole can only exist on the surface $\xi = L$ and will therefore scale with the area of the surface as $L^2 \sim 4N^2\theta$. Moreover, such information restricted on outermost matter surface L from below can be propagated onto the Schwarzschild horizon. Quantum fluctuations of the matter fields on the matter surface L can set up sympathetic vacuum fluctuations in the overlying region $L \leq \xi \leq 2M$, which deposit information on the Schwarzschild horizon. In this sense our treatment suggests a physical picture of how it can be that the dynamical degrees of freedom (and hence the entropy of a black hole) reside on the surface, consistent with the Holographic Principle. Based on this argument it follows that, in this treatment, a black hole is holographic [38].

5 Closing discussion

In this work we have put forward a model of a black hole, set on a non-commutative background. The entire spacetime is an exact solution of the Einstein Field Equations. Our approach is based on a set of axiomatic statements we make, namely (i) a proposition that imposes a non-commutative deformation mechanism on a density field introduced on a non-commutative background into two different fields, and (ii) a lemma that points to where such fields will reside on the manifold. It is found that a de Sitter-like field is one of the

by-products of the deformation mechanism. Such a de Sitter-like field which resides in the inter-lattice (off-cell) regions is responsible for producing negative pressure that offsets singularity formation. The other product, the matter field, resides on-cell. The result is that the interior region \mathcal{R} , where the fields reside, constitutes bands of concentric spheres of thickness $\sqrt{\theta}$ alternatingly dominated by matter and de Sitter-like fields, respectively. Each matter fluid region always encloses a de Sitter-like fluid region. The alternating set-up goes all the way down to the innermost central sphere, which sphere is manifestly de Sitter-like. This arrangement results in a non-singular, quantized black hole. The approach has the advantage that the black hole density is not a function of the mass (as is the case in some recent treatments). The net density ρ_0 is, instead a fixed quantity. The approach puts an upper bound on black hole density as a universal parameter. This is one of the questions this work set up to address.

The second question, regarding the stress-energy tensor has also been addressed. It is shown that the negative pressure (at various specific values of ξ) responsible for halting the formation of a curvature singularity is produced through the non-commutative deformation mechanism which we proposed. The choice of the equation of state and hence the choice of the stress-energy tensor is justified only by the the proposed mechanism. We found, however, that across the region \mathcal{R} occupied by the matter fields the average value of each of the pressure elements in the stress-energy tensor is a vanishing quantity. The mechanism conserves the original energy in the mass M . Interestingly, this result also implies that the average value of the stress-energy tensor of this spacetime (and hence the bulk of matter in the interior black hole region \mathcal{R}) satisfies all the classical energy conditions of GR, while individual specific surfaces in this region do not. The mixed result is important to the extent that it demonstrates locally quantum characteristics of the fields due to a non-commutative background at each surface ξ sum up to present a globally classical view.

The spacetime proposed has an interior central region \mathcal{R} of size L where the fields reside. The region \mathcal{R} is bounded from above by an interior vacuum region $L < \xi \leq 2M$. The full sub-space $0 \leq \xi \leq 2M$ is a Kantowski-Sachs spacetime. On the other hand, the region $2M < \xi \leq \infty$ is still the asymptotically flat Schwarzschild spacetime. The entire spacetime is described by a well-behaved metric (continuous with continuous derivatives) satisfying the Israel junction conditions. In particular, the matter-vacuum interface $\xi = L$, is manifestly well-behaved.

We have found that a black hole described on a non-commutative background presents several new features and suggests physically motivated intuitive interpretations to some others already known. In particular, we showed that a black hole defines its own size scale $L = L(M)$ of the interior region it occupies \mathcal{R} . This region is quantized in terms of $\sqrt{\theta}$ as $L = 2N\sqrt{\theta}$, where $N = 1, 2, 3..$ and $\sqrt{\theta}$ is the non-commutative length scale (corresponding to the cell size), which in this treatment turns out to be the size scale of each spherical band. Finally, we pointed out that the matter interior of the black hole in this work is frozen, with no existing dynamical degrees of freedom. Such degrees of freedom are shown, with justification, to exist on the surface instead. In our view this creates an intuitive basis for why the entropy scales with surface area of the horizon.

In summary we find that a black hole defined on a non-commutative background is necessarily both non-singular, quantized and holographic. A discussion of the horizon structure,

radiation process and stability of this spacetime will be reported elsewhere.

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